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Neutrino Trapping in a Supernova and Ion Screening

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Abstract

Neutrino-nucleus elastic scattering is reduced in dense matter because of correlations between ions. The static structure factor for a plasma of electrons and ions is calculated from Monte Carlo simulations and parameterized with a least squares fit. Our results imply a large increase in the neutrino mean free path. This strongly limits the trapping of neutrinos in a supernova by coherent neutral current interactions.

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A (core collapse) supernova radiates large numbers of neutrinos. Indeed, the energy in neutrinos is 100 times greater then that in all other forms of matter [1]. Therefore, supernova models may depend on the details of neutrino interactions in dense matter. In this paper, we calculate how correlations in the medium modify the important neutrino-nucleus elastic scattering cross section. This cross section is large because it involves coherent scattering from all of the nucleons in a nucleus [2]. However, when the neutrino wave length is comparable to the inter particle spacing there are also coherent contributions from different nuclei. These can screen the interaction and lead to a large reduction in the cross section.

In the present supernova model, the core of a massive star runs out of nuclear fuel and collapses [3]. This core is composed of a dense plasma of electrons and nuclei. As the density reaches 10^{11} to 10^{12} g/cm^3 the medium starts to become opaque to neutrinos. The neutrino opacity is thought to be dominated by neutrino-nucleus elastic scattering (as long as a significant fraction of the matter is in nuclei). This opacity insures that neutrino transport involves diffusion (rather then free streaming). The diffusion time can become long compared to the dynamical time scale, thus trapping neutrinos and their lepton number.

The neutrino-nucleus elastic cross section in free space is [4],

$$d\sigma_0/d\Omega = \frac{G^2C^2E_\nu^2(1+\cos\theta)}{4\pi^2}, \tag{1}$$

with G the Fermi constant, E_ν the neutrino energy, θ the scattering angle and the total weak charge C of a nucleus of charge Z and neutron number N is

$$C = -2Z\sin^2\Theta_W + (Z-N)/2. \tag{2}$$

(We use a Weinberg angle of $\sin^2\Theta_W = .223$.) In a dense plasma this cross section is modified by electron [5,6] and ion [6] screening. Imagine a single impurity ion in a dense plasma. Extra electrons will be attracted to the impurity. Since these electrons have weak interactions they screen both the electro-magnetic and weak charge of the ion. However, the very dense relativistic electron gas is quite rigid because of the large Fermi momentum. This limits the effect of electron screening (see below).

Other ions can also screen the impurity by creating a small hole in the ion distribution. At temperatures of order one MeV, the ions are essentially classical and their screening is not impeded by a large Fermi energy. Therefore, we will focus on ion screening in this paper. Some ion screening results have been presented in Ref. [6]. Here we calculate screening for a broad range of densities and determine its impact on the neutrino mean free path. We also provide a parameterization of our results. This, or a further simplification, will allow the incorporation of screening in neutrino transport codes.

Ion screening is included by multiplying Eq. (1) by the static structure factor S_q of the ions [7],

$$d\sigma/d\Omega = d\sigma_0/d\Omega\, S_q. \tag{3}$$

Here q is the momentum transfer and $d\sigma/d\Omega$ the effective cross section in the medium. We discuss S_q below.

The transport cross section is the angle integral of Eq. (3) with a factor of $(1-\cos\theta)$,

$$\sigma^t = \int d\sigma/d\Omega\,(1-\cos)\,d\Omega \ = \sigma_0^t \ < S > . \tag{4}$$

The free transport cross section is, $\sigma_0^t = \frac{2}{3}G^2C^2E_\nu^2/\pi$, and $<S>$ is the angle average of S_q,

$$<S>= \frac{3}{4}\int_{-1}^{1} d\cos\theta\,(1+\cos\theta)(1-\cos\theta)\,\mathrm{S}_{\mathrm{q}(\theta)}. \tag{5}$$

Here $(1+\cos\theta)$ is from the angular dependence of the free cross section and $q(\theta)^2 = 2E_\nu^2(1-\cos\theta)$. Thus, ion screening can be incorporated into neutrino transport codes be multiplying the existing interactions by the factor $<S>$ [1]. This depends on the density, temperature and neutrino energy. The transport mean free path λ then follows, $\lambda = 1/(n\sigma^t)$, with n the number density of ions.

The static structure factor S_q is determined from a Monte Carlo simulation [8] of the radial distribution function $g(r)$ [9]. This gives the probability to find another ion a distance r from a given ion and is calculated by histograming the relative distances in the simulation [8],

$$S_q = 1 + n\int d^3r\,\mathrm{e}^{-i\mathbf{q}\cdot\mathbf{r}}(g(r)-1). \tag{6}$$

Equations (5,6) yield a simple integral for $<S>$,

$$<S>= 1 + \frac{4\pi n}{E_\nu^2}\int_0^\infty dr f(2E_\nu r)(g(r)-1), \tag{7}$$

with

$$f(x) = 72(\cos x + x\sin x - 1)/x^4 - 6(5\cos x + x\sin x + 1)/x^2. \tag{8}$$

The classical canonical partition function is simulated using $N_i \approx 1000$ ions in a box of volume $V = N_i/n$ with periodic boundary conditions. The ions interact via screened Coulomb potentials,

$$v(r) = \frac{Z^2e^2}{4\pi r}\mathrm{e}^{-r/\lambda_e}. \tag{9}$$

Here $\lambda_e = \pi/(ek_F)$ describes the electron screening of the ion-ion interaction [10]. Note, this Yukawa approximation can be replaced by a more accurate description at high momentum transfers. However, we are primarily interested in momentum transfers q much less then the Fermi momentum $q << k_F$. Therefore Eq. (9) should be adequate for our purposes.

The system is warmed up for about 200 Metropolis sweeps starting from either a simple cubic lattice or a uniform distribution. Statistics are then accumulated using 400 configurations each of which is separated by 5 sweeps. This yields S_q with a typical statistical accuracy of $2-3\times10^{-3}$. These results are close to S_q for a pure one component plasma [11].

We parameterize our Monte Carlo results for $<S>$ as a function of two dimension-less variables. It is a strong function of

$$\bar{E} = E_\nu a \tag{10}$$

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[1]Note, Eqs. (1,4) have ignored axial current contributions to the cross section. These may be significant when $<S>$ is very small.

Here, the ion sphere radius a measures the average distance between ions [9],

$$a = [3/(4\pi n)]^{1/3}. \tag{11}$$

Next, $< S >$ is a weak function of Γ which characterizes the strength of the interaction. This is the ratio of a typical Coulomb potential to the thermal energy kT [9],

$$\Gamma = \frac{Z^2 e^2}{4\pi a kT}, \tag{12}$$

(with $e^2 = 4\pi\alpha \approx 0.0917$). In general $< S >$ is a function of the density and temperature separately. However, if one ignores the relatively small effect of the screening length λ_e in Eq. (9) then $< S >$ only depends on Γ (and $\bar{E}$). We have performed simulations for a pure ^{56}Fe plasma at $kT = 1$ MeV. We assume results can be extrapolated to other compositions and temperatures by calculating the appropriate Γ.

A least squares fit of our Monte Carlo results valid for all E_ν and $1 < \Gamma < 150$ is carried out. This fit is based on simulations for twelve values of Γ between 0.87 and 151.8. For a temperature of one MeV this corresponds to ^{56}Fe densities from 2×10^7 to 9×10^{13} g/cm^3. We approximate $< S >$,

$$< S(\bar{E}, \Gamma) >= 1/[1 + \exp(-\sum_{i=0}^{6} \beta_i(\Gamma)\bar{E}^i)], \tag{13}$$

for

$$\bar{E} < E^*(\Gamma) = 3 + 4/\Gamma^{1/2}. \tag{14}$$

While for $\bar{E} > E^*$ we assume,

$$< S(\bar{E}, \Gamma) >= 1. \tag{15}$$

The coefficient functions $\beta_i(\Gamma)$, for i=3,4,5 and 6 are expanded in a power series in $\Gamma^{1/2}$,

$$\beta_i(\Gamma) = \beta_{i1} + \beta_{i2}\Gamma^{1/2} + \beta_{i3}\Gamma + \beta_{i4}\Gamma^{3/2}. \tag{16}$$

The coefficients β_{ij} are collected in table I. Finite size effects contaminate the Monte Carlo results for small $\bar{E}$. Therefore we use RPA results for β_0,

$$\beta_0 = \ln[0.300/(0.300 + 3\Gamma)], \tag{17}$$

$\beta_1 = 0$ and $\beta_2 = 6.667$.

The error in the fit is typically less then 0.01. Although, for very large Γ, $< S >$ oscillates around one at large $\bar{E}$. This oscillation is not reproduced by our fit and can lead to an error as large as 0.05. However, this only occurs at very high densities and is expected to have negligible impact on the dynamics. Again, the fit is valid for all neutrino energies and $1 < \Gamma < 150$. For smaller Γ a good estimate is provided by simply setting $\Gamma = 1$. (Note, here $< S >$ is only important at very small neutrino energies.) Likewise, for $\Gamma > 150$ a reasonable estimate is provided by setting $\Gamma = 150$ (as long as the system is in the liquid phase). A solid is expected to form for $\Gamma \approx 180$ [12]. This may be relevant for models of

type Ia supernovae [13]. The very interesting problem of “Bragg diffraction” of neutrinos in a radioactive crystal remains to be investigated. Neutrino wave lengths can be comparable to the lattice spacing.

We use this fit for $< S >$ to calculate the mean free path of a neutrino in a plasma of ions, neutrons and electrons. For example, Cooperstein and Wambach [14] modeled matter at 10^{12} g/cm^3 as consisting of $X_n = 6$ percent free neutrons and 94 percent nuclei of average charge $Z \approx 37$ and average mass $A \approx 97$ at a temperature of 1.5 MeV. This is appropriate for the collapse phase of a supernova. We use this composition in calculating the mean free path. For simplicity, the composition and temperature are assumed not to change with density and we ignore the strong interactions between ions and or neutrons.

The transport mean free path λ is assumed dominated by elastic scattering off of nuclei and neutrons [15]

$$\lambda = \frac{15 \text{ km}}{\rho_{12}} \Big(\frac{10 \text{ MeV}}{E_\nu}\Big)^2 [(1 - X_n)\frac{C^2}{A} < S > R_e + X_n(c_v^{n2} + 5c_a^{n2})]^{-1}. \tag{18}$$

Here ρ_{12} is the density in units of 10^{12} g/cm^3, the weak couplings of a neutron c_i^n are given in ref. [16] and R_e is an additional correction factor that describes electron screening. This is calculated in ref. [6] and can be approximated, see below Eq. (9),

$$R_e \approx \left[1 + (\frac{c_v^e Z}{C})\frac{1}{1 + 2.5E_\nu^2\lambda_e^2}\right]^2. \tag{19}$$

Each ion has an electron cloud around it. Electron neutrinos or anti-neutrinos couple to this with strength, $c_v^e = 2\sin^2\Theta_W + \frac{1}{2}$, while muon neutrinos do not see the electron cloud $c_v^e \approx 0$ and thus $R_e \approx 1$.

The mean free path λ is shown in Fig. 1. To our knowledge, almost all present supernova simulations use Eq. (18) with $< S >= R_e = 1$. This leads to a very small mean free path (which traps neutrinos for densities of about $\rho_{12} = 0.5$ and above). However, including $< S >$ leads to a dramatic increase in λ and to a large change in its density dependence. The rapid decrease of $< S >$ with density can lead to a λ which actually *increases* with density. Over a range of densities λ for $E_\nu = 10$ MeV is greater then 10 km. This is much larger then the unscreened λ (≈ 0.4 km at $\rho_{12} = 5$). Finally, electron screening causes λ for a ν_e to be about 15 percent *larger* then for a ν_μ.

Screening effects are even more important for lower E_ν. For example at 5 MeV, λ is greater then 45 km even at $\rho_{12} = 10$. This is larger then the size of the dense system (≈ 30 km) so a neutrino-sphere may not form at all (for this energy).

Figure 1 shows that λ *is larger then the size of the system for E_ν less then or equal to about 7.5 MeV*. For E_ν between 7.5 and about 10 MeV the relatively large λ will allow neutrinos to diffuse out of the system (in about a msec or less). These are main results of this paper.

However, at $E_\nu = 20$ MeV (or above) screening is reduced and the overall $1/E^2$ scale of λ is smaller so that the mean free path is significantly shorter. Screening is not very sensitive to temperature (as long as there are no large changes in composition). Changing T leads to a change in Γ see Eq. (12). However $< S >$ is not a strong function of Γ. Likewise $< S >$ is not very sensitive to the average Z of the material. Changes in the average A change a in Eq. (10) and the overall factor C^2/A in Eq. (18). Thus λ decreases with increasing A.

The static structure factor S_q describes the total strength to scatter from the medium. At high density, this may be dominated by the excitation of *ion* plasma oscillations rather then (quasi-) elastic scattering. These plasma osc. have energies $\omega \approx (Z^2e^2n/M_i)^{1/2}$ (with ion mass M_i) and may lead to a much larger net neutrino energy loss per $\nu - A$ collision [17]. This can impact neutrino thermalization and heating.

Screening effects will be all but absent after the supernova shock wave dissociates the nuclei. Then λ will be relatively short because of scattering from large numbers of nearly free neutrons and protons. Thus, *the neutrino opacity is small (because of screening) before the shock wave and large afterwards.* This change in opacity could have important effects on the dynamics. Perhaps the situation is not unlike the photon opacity of the universe being large before and small after recombination.

We now speculate on some of the implications of screening on supernova simulations. We emphasize that final conclusions await neutrino transport calculations. First, more neutrinos may escape allowing additional electron capture (as escape reduces the neutrino chemical potential). Electron capture reduces the pressure and the energy of the shock.

For example a 20 MeV (or higher) electron capture neutrino produced at $\rho_{12} = 5$ can have its energy reduced to about 10 MeV in a time of order 1/3 msec by electron scattering. The neutrino can then diffuse out of the core in about a msec. Alternatively the neutrino's energy can be reduced to 7.5 MeV (in about a msec) and then directly escape.

Second, increased diffusion should raise the neutrino luminosity during the early stages of a supernova. [Perhaps, this could be observable if many prompt ν_es are detected from a nearby supernova [1].] This may enhance the neutrino transport of energy to the shock. Screening has almost no effect on the opacity of low density matter or of the dissociated material after shock passage. Thus screening should not interfere with the ability of material near the shock to absorb energy from neutrinos. (Note, screening introduces a new density dependence to the neutrino interactions.)

We have calculated the effect of ion screening on neutrino-nucleus elastic scattering. Our Monte Carlo results for the angle average of the static structure factor have been fitted to an analytic formula. This may allow the inclusion of screening in simulations. We find that the mean free path of a 10 MeV (or lower) neutrino is greatly increased. This could have important effects on the early stages of a supernova.

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TABLES

TABLE I. Parameters β_{ij} from a least squares fit of the angle averaged static structure factor $< S >$, see text.

Coeff.	$j = 1$	2	3	4
β_{3j}	-7.362056	0.5371365	-0.1078845	4.189612E-3
β_{4j}	3.4489581	-0.40251656	9.0877878E-2	-3.4353581E-3
β_{5j}	-0.74128645	0.11019855	-2.5359361E-2	9.0487744E-4
β_{6j}	5.9573285E-2	-1.0186552E-2	2.2791369E-3	-7.4614597E-5

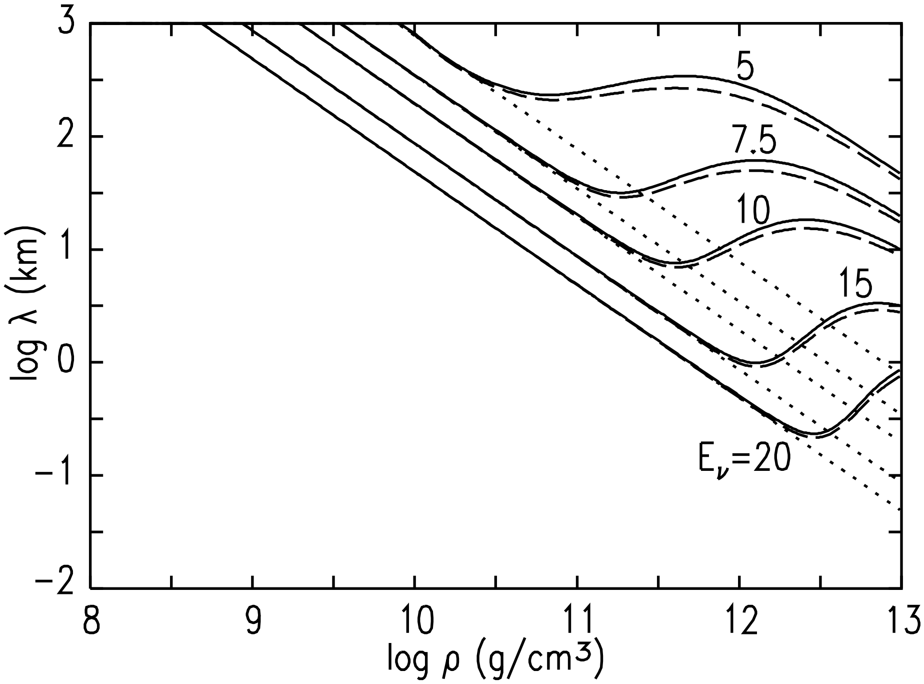

FIG. 1. Neutrino transport mean free path vs. density. The solid lines include both ion $< S > \neq 1$ and electron $R_e \neq 1$ screening and are appropriate for ν_e, $\bar{\nu}_e$ while the dashed lines for ν_μ neglect electron screening $R_e = 1$. Finally the dotted lines (used in most present supernova simulations) neglect all screening $< S >= R_e = 1$. Top to bottom, the curves are for neutrino energies of $E_\nu = 5$, 7.5, 10, 15, and 20 MeV.